INVARIANT GEOMETRIC REPRESENTATION OF 3D POINT CLOUDS FOR REGISTRATION AND MATCHING

Soma Biswas, Gaurav Aggarwal and Rama Chellappa

Center for Automation research
University of Maryland
College Park, MD 20742

ABSTRACT

Though implicit representations of surfaces have often been used for various computer graphics tasks like modeling and morphing of objects, it has rarely been used for registration and matching of 3D point clouds. Unlike in graphics, where the goal is precise reconstruction, we use isosurfaces to derive a smooth and approximate representation of the underlying point cloud which helps in generalization. Implicit surfaces are generated using a variational interpolation technique. Implicit function values on a set of concentric spheres around the 3D point cloud of object are used as features for matching. Geometric-invariance is achieved by decomposing implicit values based feature set into various spherical harmonics. The decomposition provides a compact representation of 3D point clouds while achieving rotation invariance.

1. INTRODUCTION

In this paper, we address the problem of registration and recognition of 3D point clouds i.e., given a pair of 3D point clouds we should be able to estimate the similarity of the two and also find the rigid transformation that relates the two in case they are reasonably similar. An ideal matching and registration algorithm should probably have following important characteristics: 1) It should be able to match and register point clouds separated by all possible rigid transformations. 2) The approach should not depend on point-to-point correspondence between the two sets. More often than not such a correspondence does not exist and even if it does, it is usually unknown and difficult to estimate directly. 3) The method should degrade gracefully in the presence of outliers or in the event of missing data. 4) The method should be able to both generalize and maintain discriminability at the same time.

With these goals in mind, we believe that representation is one of the most critical issues. The representation should not only describe the object well but also generalize and extend seamlessly to perform robust recognition and registration. In this paper, we use implicit representation to achieve these goals. We generate implicit surfaces from a 3D point cloud using a thin-plate like variational interpolation technique [1]. The approach represents the surface with the help of radial basis functions defined around a bunch of pivots sampled on concentric spheres around the object. The parameters are estimated by solving a system of linear equations where constraints are provided by the input point cloud and a few carefully but efficiently chosen exterior points. The estimated parameters form a signature of the input point cloud.

Given two such signatures, explicitly solving for the optimal alignment to measure the similarity of the two is impractical. Though variations in scale and translation are handled using traditional normalization methods, such methods for rotation are less robust thereby affecting the matching process. Spherical harmonic representation of spherical functions provides a very efficient and robust method to achieve rotation invariance. We decompose the implicit function values on a set of concentric spheres around the point cloud into various spherical harmonics. The energies at various harmonic levels do not change with rotation which forms the rotation invariant representation of the given point cloud.

1.1. Previous Work

Given the importance of automated object recognition systems, the problem of shape matching and registration has been an active area of research for the past few years. Matching models across similarity transformations has been one of the main challenges in this area. Traditionally, this has been handled either by finding a canonical transformation for each model or by characterizing models with invariant descriptors or by explicitly solving for the optimal transformation.

Shape histograms [2], shape distributions [3], Extended Gaussian Images [4], wavelets [5], higher order moments [6] etc. are a few of the descriptors which have been explored to describe 3D shapes. Other than a few histogram based features which are invariant to geometric transformations, the descriptors are normalized by using the center of mass for translation, standard deviation for scale and principal axes based alignment for rotation. Though translation and scale normalizations perform reasonably well, PCA-normalization falls short of providing a robust alignment [7]. To this end, Kazhdan et al. [7] propose a spherical harmonic representation of such descriptors to achieve rotation invariance. In this paper, we also use spherical harmonics for rotation invariance but we apply it on implicit values based feature vector which, as we show, is a more complete and precise representation.

The alternative approach involves explicitly solving for optimal transformation using registration methods like Iterative Closest Point Matching (ICP) [8] [9], Generalized Hough Transform [10], Geometric Hashing [11], etc. before computing the similarity of the models. Such approaches can be quite inefficient in a database retrieval kind of application as one will need to register every query model with all the models in the database (assuming the algorithm is able to register models correctly across large transformations).

Given the advantages implicit representation provides, implicit surface generation has been an important area of research in Computer Graphics. The book by Bloomenthal et al. [12] provides an excellent overview of the area. Most of the methods define implicit surfaces in the form of quadrics, blobs or radial basis functions around the input 3D points. Almost all of them assume that a polygonal mesh connecting the 3D point cloud is given as input. In contrast, we generate implicit surface using only 3D point clouds.

The rest of the paper is organized as follows: The next section describes the approach for generating implicit surfaces. In Section 3, we describe the use of spherical harmonic decomposition to obtain rotation invariant feature vector. Section 4 describes a simple registration method using the generated isosurfaces. Results are shown to reflect the efficacy of the approach for the tasks of matching and registration.

2. IMPLICIT REPRESENTATION OF A SURFACE

We use implicit surfaces based on a variational interpolation technique which is a generalization of thin-plate interpolation. The method is similar to the one proposed by Yngve et al. [1] with two main differences:1) We do not use the polygonal mesh information and generate implicit surfaces using just the 3D point cloud. 2) We use uniformly sampled points on concentric spheres as the pivot points instead of choosing them adaptively in an iterative fashion as done in [1]. This provides us with a globally unique representation of the object. The drawback is that this can prevent us from getting a very precise representation of the object but that is not the goal here as opposed to [1] where accurate reconstruction is the main objective. The approximate isosurface, we obtain, helps in generalization while not losing discriminable characteristics. These are the kind of properties one desires from a representation for the task of matching.

Generation of variational implicit surfaces involves solving a scattered data interpolation problem [13]. To create a variational implicit function, one needs to choose a certain number of constraint points $\{x_1, x_2, \ldots, x_n\}$, along with a set of implicit function values $\{h_1, h_2, \ldots, h_n\}$ at the given constraint positions. Typically, there are three types of constraints: 1) *Boundary constraints* are those constraint points which lie on the surface and take the value zero. 2) The *interior constraints* lie inside the surface represented by the point cloud and are given positive values. 3) The *exterior constraints* lie outside the surface and are assigned negative values. We use an implicit function of the form:

$$f(x) = \sum_{j=1}^{k} d_j \phi(x - c_j) + P(x) \tag{1}$$

Here c_j are the locations of the pivots, d_j are the weights which we need to estimate, $P(x)$ is a first degree polynomial to account for the linear and constant portions of the implicit function f.

As we deal with point sets which represent object surfaces, the implicit functions should be chosen to make the surfaces reasonably smooth. The smoothness is also useful in making the representation fairly robust to outliers. Therefore, we take $\phi(x) = \| x \|^3$ as this function minimizes the curvature functional $\int_{x\epsilon\Omega} \sum_{i,j} \left(\frac{\partial^2 f(x)}{\partial x_i \partial x_j}\right)^2 dx$ [14].

Given a set of 3D points, we solve for the weights d_j and coefficients of the polynomial $P(x)$ using the following linear constraints:

$$h_i = \sum_{j=1}^{k} d_j \phi(x_i - c_j) + P(x_i) \tag{2}$$

These equations being linear with respect to d_j and the coefficients of $P(x)$, can be formulated as

$$\begin{pmatrix} \phi_{11} & \phi_{12} & \ldots & \phi_{1k} & 1 & x_1^x & x_1^y & x_1^z \\ \phi_{21} & \phi_{22} & \ldots & \phi_{2k} & 1 & x_2^x & x_2^y & x_2^z \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi_{n1} & \phi_{n2} & \ldots & \phi_{nk} & 1 & x_n^x & x_n^y & x_n^z \\ 1 & 1 & \ldots & 1 & 0 & 0 & 0 & 0 \\ c_1^x & c_2^x & \ldots & c_k^x & 0 & 0 & 0 & 0 \\ c_1^y & c_2^y & \ldots & c_k^y & 0 & 0 & 0 & 0 \\ c_1^z & c_2^z & \ldots & c_k^z & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_k \\ p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix}$$

$$= \begin{pmatrix} h_1 & h_2 & \ldots & h_n 0 & 0 & 0 & 0 \end{pmatrix}^T \tag{3}$$

The pivot points c_j are sampled uniformly on a bunch of concentric spheres around the center of mass of the object. As they are fixed irrespective of the object, we get a globally unique and compact representation of the 3D point cloud in the form of the parameter vector (containing d's and p's) obtained by solving the system in (3).

We use all the input 3D points to generate linear constraints of the form (2) with zero as the implicit function value. As we do not use the polygon information, it is not easy to identify the points which lie inside the object with certainty (if the object is not convex). In comparison, choosing exterior points is much easier even without any polygonal information. We envelop the point cloud with a tight fitting ellipsoid with the axes of the ellipsoid aligned in the direction of the principal components of the distribution of the 3D points. Points are sampled on the enveloping ellipsoid to get the exterior constraints. The points on the ellipsoid which lie inside the convex hull of the 3D point cloud are not considered. The negative of the distance of each exterior point from the closest point in the point cloud is assigned as the implicit function value to get the linear constraints of the form (2).

Though one can come up with more complex and iterative strategies [1] to get a better reconstruction but as mentioned before that is not our goal. We aim at generating a smooth and approximate isosurface for a given object which is representative of its class. Fig. 1 shows a few isosurfaces generated using this approach. It is worthwhile to note that only point cloud information is used to generate these surfaces. In contrast, most state-of-the-art graphics approaches use polygonal or volumetric information to generate isosurfaces.

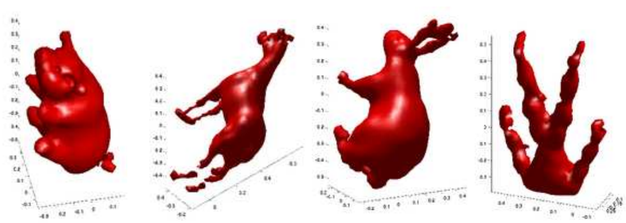

Fig. 1. A few examples of the generated isosurfaces. All the surfaces were generated using just 500 pivot points.

3. ROTATION INVARIANT REPRESENTATION

Though the choice of the same set of pivots to model any point cloud provides us with a unique representation, the estimated parameters are not invariant to similarity transformations of the object. Thus two point clouds cannot be directly compared based on their estimated parameter values.

Spherical harmonic decomposition of a spherical function provides a very simple and efficient way to obtain a rotation invariant representation (though not lossless) of the function. The approach proposed in the previous section provides a very efficient method to generate approximate smooth isosurfaces of given 3D point clouds which as shown in Fig. 1, can help generalize without losing discriminability. Therefore, instead of extracting some generic feature and using spherical harmonic transformation, we intend to use the isosurfaces generated from 3D point clouds to obtain the rotation-invariant descriptor. To this end, we propose using implicit function values as the spherical feature. We compute implicit function values using (1) at uniformly sampled points on a set of concentric spheres around the object. The spherical harmonic decomposition of these implicit values based spherical functions is then computed for each sphere to get the rotation invariant signature as described below.

If the implicit values based function can be represented in terms of spherical harmonics as,

$$f(\theta, \phi) = \sum_{l=0}^{k} \sum_{m=-l}^{l} a_{lm} Y_l^m(\theta, \phi) \tag{4}$$

then the norms, $\sqrt{\sum_{-l \leq m \leq l} |a_{lm}|^2} \quad \forall \quad 0 \leq l < k$, are invariant to rotation. These rotation invariant energies at different harmonic levels for all the concentric spheres are used to form the desired rotation invariant feature vector. Quite clearly, larger the value of k is, the better the function can be represented using the spherical harmonics. In our experiments, we set k to 64. Fig. 2 shows the ability of the proposed feature vector to measure similarity of various models (from The Princeton Shape Benchmark [15]) reliably across changes in scale and rotation.

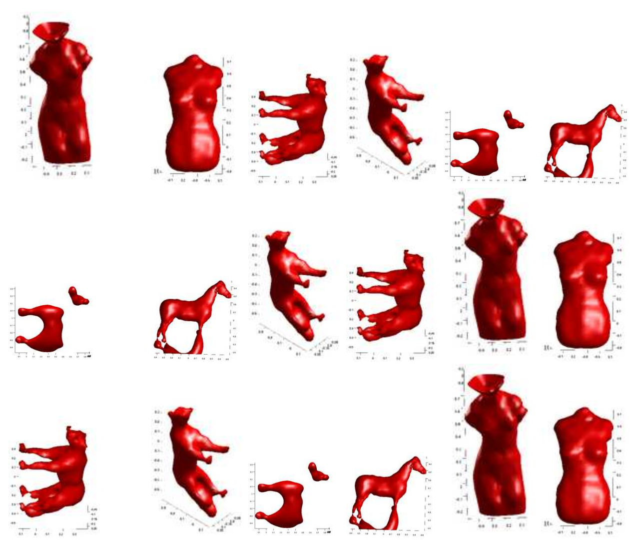

Fig. 2. The figure displays the ability of the proposed feature vector to match objects across geometric transformations. In each row, the models are arranged in the order of decreasing estimated similarity w.r.t. the leftmost model in the row.

4. ESTIMATION OF THE RIGID TRANSFORMATION USING ISOSURFACES

The proposed rotation invariant descriptor helps in matching 3D point clouds across arbitrary rotations. In this regard, spherical harmonic decomposition for handling rotation in 3D is analogous to that of Fourier decomposition for handling translation. Though the phase information in Fourier decomposition is useful in estimating the translation, spherical harmonics can not directly be used for estimating the underlying rotation. Therefore, we propose a simple and effective strategy to estimate the underlying rotation between two reasonably similar objects using their generated isosurfaces.

The approach makes use of the intuition that correct rotation will make a point cloud satisfy the implicit function (Equation (1)) of the other at most points and vice-versa i.e., $R' = R$ minimizes the following implicit function value for all points if the correct underlying rotation is R:

$$f(x_i^R) = \sum_{j=1}^{k} d_j \phi(x_i^R - R^{'}(c_j)) + P((R^{'})^{-1}(x_i^R)) \quad (5)$$

where x_i^R are the points of the rotated object, c_j are the pivots of the base object while d_j and P are the estimated isosurface parameters of the base object and R' is the rotation matrix corresponding to the hypothesized rotation. The optimization to obtain the optimal R is done using $lsqnonlin$ function in MATLAB. Translation and scale variations are taken care of using the traditional normalizations. Fig. 3 shows an example of registering two animals using this approach. It is worthwhile to note that the method is able to cope with small misalignment of the center-of-mass of the objects as can be seen in the shown example. In addition, unlike ICP [8], the proposed approach performs registration without explicitly solving for correspondence.

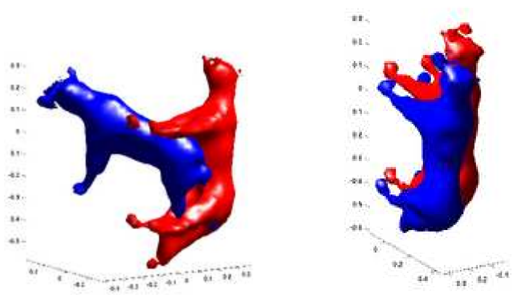

Fig. 3. Registration result: The figure shows two models before and after registration using the proposed approach.

5. SUMMARY AND DISCUSSION

We proposed an isosurface-based approach to register and match 3D point clouds. The approach generates smooth implicit representations of point clouds to achieve the desired goals. The algorithm is quite efficient and is therefore suitable for database retrieval applications. The approach is quite adaptive as far as generalization-discriminability trade-off is concerned.

6. REFERENCES

[1] Yngve G. and G. Turk, "Robust creation of implicit surfaces from polygonal meshes," *IEEE Transactions on Visualization and Computer Graphics*, vol. 8, no. 4, pp. 346–359, October-December 2002.

[2] M. Ankerst, G. Kastenmuller, H. P. Kriegel, and T. Seidl, "3d shape histograms for similarity search and classification in spatial databases.," in *SSD*, 1999.

[3] R. Osada, T. Funkhouser, B. Chazelle, and D. Dobkin, "Matching 3d models with shape distributions," *Shape Matching International*, pp. 154–166, 2001.

[4] B.K.P. Horn, "Extended gaussian images," *PIEEE*, vol. 72, no. 12, pp. 1656–1678, December 1984.

[5] J. Gain and J. Scott, "Fast polygon mesh querying by example," in *SIGGRAPH '99: ACM SIGGRAPH 99 Conference abstracts and applications*, 1999.

[6] M. Elad, A. Tal, and S. Ar, "Content based retrieval of vrml objects: an iterative and interactive approach," in *Proceedings of the sixth Eurographics workshop on Multimedia 2001*, 2002, pp. 97–108.

[7] M. Kazhdan, T. Funkhouser, and Rusinkiewicz S., "Rotation invariant spherical harmonic representation of 3d shape descriptors," in *Eurographics Symposium on Geometry Processing*, 2003.

[8] P. Besl and N. McKay, "A method for registration of 3d shapes," *IEEE Transactions on PAMI*, pp. 239–256, 1992.

[9] Z. Zhang, "Iterative point matching for registration of free-form curves and surfaces," *IJCV*, pp. 119–152, 1994.

[10] D. Ballard, "Generalized hough transform to detect arbitrary patterns," *IEEE Transactions on PAMI*, pp. 111–122, 1981.

[11] Y. Lamdan and H. Wolfson, "Geometric hashing: A general and efficient model-based recognition scheme," 1988, pp. 238–249.

[12] J. Bloomenthal, ed. Morgan Kauffman, "Introduction to implicit surfaces," 1997.

[13] G. Turk and J. F. O'Brien, "Variational implicit surfaces," *Technical Report 15, Georgia Institute of Technology*, 1999.

[14] J. Duchon, "Spline minimizing rotation-invariant semi-norms in sobolev spaces," *Lecture Notes in Math*, vol. 571, pp. 85–100, 1976.

[15] P. Shilane, P. Min, M. Kazhdan, and T. Funkhouser, "The princeton shape benchmark," *Shape Modeling International, Genova, Italy*, June 2004.