Active Damping of Filter Resonance Using Resonant Integrator Based Filter

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Abstract

Use of LCL filter along with a Voltage Source Inverter (VSI) requires damping of resonance. Active Damping (AD) methods are preferable compared to Passive Damping (PD) methods from efficiency point of view. Recent researches show that achieving stable operation of a grid connected VSI with LCL filter at a low sampling frequency to resonance frequency ratio can cause instability. In this paper a Resonant Integrator (RI) based second order filter has been proposed for AD of LCL filter resonance for stable operation at low sampling frequency to resonant frequency ratio. The method extracts resonant frequency capacitor current from sensed capacitor voltage in a per phase manner and does not need any additional sensors. Experimental results have been included in support of the theoretical analysis.

Index Terms

Active damping, Resonant integrator, LCL filter, Filter resonance.

I. INTRODUCTION

Use of LC or LCL filter at the AC terminals of VSI based applications, such as grid connected converter, has become a common practice. This is to achieve a smooth output voltage waveform and lower injection of switching frequency component of current [1]–[4]. This also requires provision for adequate damping at the resonant frequency. Traditionally this resonance is damped by using hardware circuits with resistance which is known as Passive Damping (PD). An alternate method where the VSI is controlled such that no additional resistive element is necessary for damping of resonance, is widely known as Active Damping (AD) [5]–[6]. The obvious advantage of this method is reduced power loss compared to PD at the cost of some added control complexity. When another digital filter structure is connected either in series or parallel with the current controller for AD, their combined effect determine the level of damping provided to the filter poles [6]–[8]. The response of the current control loop and damping performance under grid impedance variation are of importance for all the AD methods [5], [9], [10].

Recent literatures [11]–[13] show that achieving stable operation of grid connected VSI with AD of LCL filter resonance at low sampling frequency (F_s) to resonance frequency (f_res) ratio, such as \( F_s / f_res < 6 \), is a challenging task. In [11] a capacitor current feedback AD method along with effect of aliasing component due to presence of high switching frequency component in capacitor current have been presented. A method suggested in [7] uses the capacitor current to emulate a voltage drop by multiplying a proportional constant which is also a virtual resistor type gain. However if capacitor voltage can be utilised for AD then effect of aliasing would be comparatively less as capacitor voltage would contain lesser magnitude of switching frequency component due to filtering effect. In [14] capacitor voltage is utilised to estimate resonance frequency component of current and provide AD for a motor drive application. In this method though the resonance cancellation signal is subtracted in stationary frame from every phase but co-ordinate transformation from stationary to synchronous reference frame and filtering is required for estimation of resonance frequency current. Choice of sensors for a grid connected VSI with LCL filter depends on application and voltage and current level [6], [7]. However for the purpose of protection a converter current sensor is typically present [15]. Also, for grid connected applications grid side voltage sensor is necessary for synchronization with grid and to meet guidelines related to ride-through requirement [16], [17]. In case the converter is connected to grid through an LC filter followed by a transformer then filter capacitor voltage would be available for sensing [10], [17]. For grid connected inverters a per phase basis AD solution, independent of other phases, is attractive considering non-ideal grid [8] and the need to operate under unbalanced condition. In [18], [19] a it has been reported that a current controller can provide sufficient damping to LCL resonance. However at low \( F_s / f_res \) ratio the system can suffer from instability. In [20] a RI based method has been suggested for active damping. This method requires a high bandwidth current control loop which would have limitation at low \( F_s / f_res \) ratio. An alternative method suggested in it uses multiple resonant controller for AD which again results in a complex implementation.

In this paper a resonant integrator [21]–[23] based second order filter (RISF) structure [20], [24]–[27] has been proposed which utilises the capacitor voltage to generate a damping signal. The RISF based AD method (ADRI) is shown to be capable of operating in a stable manner at low ratio of switching frequency to resonant frequency. Although the method in the current
The paper is based on RI, but it has significant differences compared to [20]. The method in present work extracts resonance frequency capacitor current information directly from sensed filter capacitor voltage as explained in the principle of operation in Section-III. Here the RISF works as a highpass filter. The proposed ADRI method provides selective damping at resonant frequency and rejects any influence of lower frequency signals due to the highpass nature of RISF. As capacitor voltage is sensed for AD instead of capacitor current so the effect of aliasing component would be lessened as presence of capacitor provides a filtering effect on switching component of current. Further, double sampling and double update mode of operation that is sampling and PWM update at both peak and valley of the PWM carrier has been adopted to reduce aliasing related problems. The study made in this paper show that the proposed active damping method is capable of working at low $\frac{f_{sw}}{f_{res}}$ ratio even under wide variation of grid inductance. The proposed method is capable of providing damping to each phase without the influence of other phases. Theoretical and experimental results have been included for both weak grid and stiff grid conditions.

II. RI BASED SECOND ORDER FILTER

The s-domain transfer function of a RI is given by $T_r = \frac{sK_{ir}}{s^2 + \omega_0^2}$. Here $K_{ir}$ is the resonant gain, $\omega_0$ and $f_0$ are the resonant frequency in rad/s and Hz respectively. A RI can be used to design a second order filter (RISF), as shown in Fig. 1(a). Here value of $\omega_0$ is selected to be the same as that of the frequency that is to be extracted from the input signal. These filters have found applications such as PLL [24], [25] and second order filter [26], [27]. Here a similar concept is applied for resonant frequency extraction to damp out the oscillation at that frequency. When the output, ‘$p’$, of RI is used to form a closed loop within the digital controller, as shown in Fig. 1(a), then the output to input relationship behaves like a band-pass filter. When multiplied with feedback gain $K_f$, shown in Fig. 1(a), the output ‘$p_1$’ at $s = j\omega_0$ equals the input ‘$u$’ in phase and magnitude.

$$\frac{P_1(s)}{U(s)} = \frac{sK_{ir}K_f}{s^2 + sK_{ir}K_f + \omega_0^2}$$

(1)
### TABLE I

PARAMETERS FOR RI BASED SECOND ORDER FILTER.

<table>
<thead>
<tr>
<th>$\zeta_{RI}$</th>
<th>$K_{ir}$</th>
<th>$K_f$</th>
<th>$\Delta \Phi$ for $\Delta f = \pm 10$ Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>100</td>
<td>11.2</td>
<td>$\pm 7.0^\circ$</td>
</tr>
<tr>
<td>0.4</td>
<td>100</td>
<td>44</td>
<td>$\pm 1.5^\circ$ [Fig. 2(b)]</td>
</tr>
<tr>
<td>0.7</td>
<td>100</td>
<td>80</td>
<td>$\pm 1.0^\circ$</td>
</tr>
</tbody>
</table>

Fig. 2. Equivalent single phase control block and circuit diagram of a 3 phase 4 wire grid interactive inverter and LCL filter with proposed ADRI and PR controller for current control.

Fig. 3. (a) Simplified system model of LCL filter and VSI at resonant frequency and (b) phasor diagram involving control signals.

\[
\frac{Q_1(s)}{U(s)} = -\frac{K_{ir} K_f \omega_0}{s^2 + s K_{ir} K_f + \omega_0^2}
\]  
\[
\frac{D_1(s)}{U(s)} = \frac{s^2 K_{ir} K_f}{s^2 + s K_{ir} K_f + \omega_0^2} \times \frac{1}{\omega_0}
\]

Transfer function $\frac{Q_1(s)}{U(s)}$ has low pass characteristics whereas $\frac{D_1(s)}{U(s)}$ has a high pass characteristics but they have same phase response, as shown in Fig. 1. When ‘$q$’ and ‘$d$’ are multiplied with feedback gain $K_f$, both $q_1$ and $d_1$ become equal in magnitude and has a phase lead of $90^\circ$ with respect to the input signal at $s = j \omega_0$. The frequency response characteristics of (1), (2) and (3) are shown in Fig. 1(b). The term ‘$K_{ir} K_f$’ appearing in the denominator of the transfer function decides the effective damping ($\zeta_{RI}$) of resonant poles of the RI. The term ‘$K_{ir} K_f$’ and ‘$\zeta_{RI}$’ are related by (4).

\[
K_{ir} K_f = 2\omega_0 \zeta_{RI}
\]

For the frequency response characteristics shown in Fig. 1(b), where $f_0 = 890$ Hz, parameters $K_{ir}$, $K_f$ and $\zeta_{RI}$ are listed in Table-I along with variation in phase response ($\Delta \Phi$) due to frequency variation ($\Delta f$) of $\pm 10$ Hz around $f_0$. In the following sections use of the transfer function $\frac{D_1(s)}{U(s)}$ for AD of LCL filter resonance has been proposed.

### III. PROPOSED METHOD OF AD

Equivalent single phase circuit diagrams along with control block architecture of a 3 phase 4 wire inverter and LCL filter with proposed ADRI method of damping is shown in Fig. 2. The proposed ADRI scheme utilises filter capacitor voltage to estimate the resonance frequency current flowing through capacitor. The concept can be understood from the simplified circuit diagram, shown in Fig. 3(a), and the phasor diagram shown in Fig. 3(b). The capacitor voltage would contain resonance frequency component of voltage $\tilde{v}_c$. If the capacitor voltage is given as input to the RI based filter whose resonant frequency $\omega_0$ is chosen such that it equals the resonance frequency of the LCL filter, then ideally $p_1$ would be equal in magnitude and in phase with $\tilde{v}_c$, as shown in Fig. 3(b). Also $d_1$ would contain only the above resonance frequency component. The resonance
frequency component would be equal in magnitude and 90° leading to \( p_1 \), as shown in Fig. 3(b). The phasor diagram of Fig. 3(b) is of ideal system where effect of discretization and system delay is not incorporated. If \( d_1 \) is multiplied with \( \omega_0 C_f \) then this would give the resonance frequency component of capacitor current \( \tilde{i}_c \), as shown in Fig. 3(b). The phasor diagram of Fig. 3(b) is of ideal system where effect of discretization and system delay is not incorporated. If \( d_1 \) is multiplied with \( \omega_0 C_f \) then this would give the resonance frequency component of capacitor current \( \tilde{i}_c \). Using this extracted current selective AD can be accomplished at the resonance frequency using the virtual resistance based approach as reported in literatures [8], [14], [28], [29]. The resultant gain, to be multiplied with \( d_1 \), is given by (5).

\[
K_g = \omega_0 C_f R_v
\]  

(5)

Next, the output of the AD block \( (m_d) \) is subtracted from \( m_c \) to provide selective damping at the resonance frequency. As \( D_1(s) \) has a high pass characteristics so the proposed method would not inject lower order harmonics present in capacitor voltage due to distortions in grid voltage waveform or burden the fundamental modulation index \( (m_c) \) generated by current controller.

IV. ANALYSIS OF PROPOSED AD

Under closed loop operation of the system shown in Fig. 2, the resonant poles of the LCL filter would change due the current loop and active damping loop. The discrete domain closed loop pole location variation can be effectively used to study the stability of the system of Fig. 2 at low \( \frac{f_{sw}}{f_{res}} \) ratio. Such an analysis can also be used to study the effect of control gains and variations in grid inductance. The discrete domain analysis of individual blocks of the grid connected closed loop VSI system of Fig. 2 are presented in this section. Further the analysis on individual blocks are combined to analyse the closed loop system shown in Fig. 2.

A. Discrete domain model of VSI and LCL filter

The discrete domain model of the inverter and LCL filter, shown in Fig. 2, can be derived from its continuous domain state space model, given by (6) and (7), by zero order hold (ZOH) method.

\[
\dot{x}_s = A_s x_s + B_i m
\]  

(6)

Where,

\[
A_s = \begin{bmatrix}
-\frac{R_1}{L_1} & 0 & -\frac{1}{L_1} \\
0 & -\frac{R_2}{L_2} & \frac{1}{L_2} \\
\frac{1}{C_f} & -\frac{1}{C_f} & 0
\end{bmatrix}
\]  

and

\[
B_i = \begin{bmatrix}
V_{dc} \\
G_{adj} \frac{2L_1}{L_2} \\
0
\end{bmatrix}
\]  

(7)

The modulation strategy for the inverter is chosen as sine triangle PWM method. The chosen states of the system are \( x_s = [i_1 \ i_2 \ v_c]^T \).

\[
x_s(k+1) = \Phi_s x_s(k) + \Gamma_i m(k)
\]  

(8)

Where,

\[
\Phi_s = e^{A_s T} \quad \text{and} \quad \Gamma_i = \int_0^T e^{A_s \eta} d\eta B_i
\]  

(9)

The discrete domain representation of this system with sampling time ‘\( T \)’ is given by (8) and (9). The ZOH method of discretization [30] inherently incorporates a delay of \( \frac{T}{2} \) sec which is equivalent to the PWM delay in an inverter [23]. The delay due to calculations inside the digital controller, which is equal to one sample time (\( T \)), can be modelled by one additional state.

\[
x_{sd}(k+1) = \Phi_{sd} x_{sd}(k) + \Gamma_{id} m(k)
\]  

(10)

\[
\Phi_{sd} = \begin{bmatrix}
\Phi_s & \Gamma_i \\
0 & 0
\end{bmatrix}
\]  

and

\[
\Gamma_{id} = \begin{bmatrix}
0 \\
1
\end{bmatrix}
\]  

(11)

The discrete domain augmented state vector is given by \( x_{sd}(k) = [i_1(k) \ i_2(k) \ v_c(k) \ m(k)]^T \). The state equations for the LCL filter system along with inverter and sampling delay are given by (10) and (11).
B. Discrete domain model of RISF

For discrete domain implementation of the block diagram shown in Fig. 1(a), the RI is discretized by two integrator based method where one integrator is implemented using Euler’s forward method and the other by Euler’s backward method [23]. This implementation has the inherent advantage of being frequency adaptive and easy to implement in a digital controller.

\[
\begin{bmatrix}
 p(k+1) \\
 q(k+1)
\end{bmatrix} = A_{kc} \begin{bmatrix}
 p(k) \\
 q(k)
\end{bmatrix} + B_{kc} u(k)
\]  
(12)

Where,

\[
A_{kc} = \begin{bmatrix}
(1 - K_{ir} K_f) & \omega_0 T \\
-\omega_0 T (1 - K_{ir} K_f) & 1 - \omega_0^2 T^2
\end{bmatrix}
\]
and \( B_{kc} = \begin{bmatrix}
K_{ir} T \\
-\omega_0 T^2 K_{ir}
\end{bmatrix} \)  
(13)

\[
y(k) = C_{kc} \begin{bmatrix}
 p(k) \\
 q(k)
\end{bmatrix} + D_{kc} u(k)
\]  
(14)

The discrete domain state space realization of the system of Fig. 1(a) can be expressed through (12), (13) and (14). Here the highpass output ‘\(d_1\)’ is used to implement AD. The difference equation for \(d_1\) can be derived from Fig. 1(a) using \(e(k)\) and \(q(k)\).

\[
y(k) = d_1(k) = K_f \frac{K_{ir}}{\omega_0} e(k) + K_f q(k+1)
\]  
(15)

However, when one computation of RISF gets over inside digital controller then updated value of ‘\(q(k)\)’ that is ‘\(q(k+1)\)’ becomes available. Therefore ‘\(d_1(k)\)’ can be computed using \(q(k+1)\), shown in (15), instead of \(q(k)\) with a small additional computational burden of one addition and one multiplication which provides improved phase characteristics of the filtering transfer function. The output matrices \(C_{kc}\) and \(D_{kc}\) are given by (16).

\[
C_{kc} = K_f \left[ \frac{-K_{ir} K_f}{\omega_0} - \omega_0 T (1 - K_{ir} K_f) \right] (1 - \omega_0^2 T^2)
\]
and \( D_{kc} = K_f \left[ \frac{K_{ir}}{\omega_0} - \omega_0 K_{ir} T^2 \right] \)  
(16)
The above model of the VSI with LCL filter and ADRI is obtained using equations (11) and (12). The term $K$ for current control, there is no feedback gain $D$. Discrete domain model of PR current controller damping of $0$ filter and ADRI is given by (17).

$C$. Discrete domain analysis of ADRI

Where, $m$ expressed by (15), is multiplied with gain $K$. The damping part of the modulation index $\Phi$ is multiplied with gain $K_g$ to generate the damping part of modulation index $m_d$ which is subtracted from $m_c$.

The damping part of the modulation index $m_d$ is given by (20). Using the relationship $m(k) = m_c(k) - m_d(k)$ and taking $m_c(k) = 0$ the closed loop characteristics matrix $\Phi_{sdc}$ can be computed as given by (21). Here $\Phi_{sdc}$ corresponds to the discrete space state matrix of the closed ADRI loop of VSI with LCL filter including delay but without the current control loop.

Here $K_g$ is the gain associated with virtual resistance which is given by (5). For different values of $R_v$ the eigenvalues of $\Phi_{sdc}$, for different values of $R_v$, between 0 to 20 Ω are shown in Fig. 4. The polar locations when $R_v = 0\Omega$ are indicated by ‘×’ symbol. Pole locations of $\Phi_{sdc}$ for $F_{sw} = 2.5kHz$ and $T = 200\mu s$ are shown in Fig. 4(a). It shows that a stable system can be obtained with ADRI for a case where $F_{sw} = 2.8$. It can also be observed from Fig. 4(a) that both LCL and RISF poles achieve almost equal level of damping of 0.2 for $R_v = 5\Omega$.

$D$. Discrete domain model of PR current controller

Apart from the proportional part ($K_{pr}$), the implementation of PR controller is similar to the RI. In case of RI implementation for current control, there is no feedback gain $K_f$ as in the RISF, shown in Fig. 1(a). The states of PR controller are $p_i$ and $q_i$. Here $e_i$ is the error input to current controller.

$$
\begin{align*}
 p_i(k+1) &= A_k \begin{bmatrix} p_i(k) \\ q_i(k) \end{bmatrix} + B_k e_i(k) \\
 q_i(k+1) &= \begin{bmatrix} 1 & \omega_0 T \\ -\omega_0 T & 1 - \omega_0^2 T^2 \end{bmatrix} \text{and } B_k = \begin{bmatrix} K_{ir} T \\ -\omega_0 T^2 K_{ir} \end{bmatrix}
\end{align*}
$$

A state space model of the RI realisation is obtained after two integrator based discretization of RI [23] and is given by (22), (23) and (24). As ‘$p_i$’ gets computed inside a digital processor so ‘$p_i(k+1)$’ would be available before actual commencement of the instant ‘$(k+1)T$’. Hence output ‘$y$’ can be computed from ‘$p_i(k+1)$’ also. This improves the phase response of the PR controller. The coefficients of output equation are $C_k = [1 \ \omega_0 T]$ and $D_k = [K_{ir} T + K_{pr}]$. 

$$
\begin{align*}
 y(k) &= C_k \begin{bmatrix} p_i(k) \\ q_i(k) \end{bmatrix} + D_k e_i(k)
\end{align*}
$$
E. Discrete domain model of the closed loop system with current control only:

The discrete domain state space model of inverter with LCL filter is given in (10) and (11). The system is of fourth order including the controller and PWM delay effects. In this subsection the closed loop system is analysed assuming that the ADRI method is not present and only current loop is present. If the two states of PR controller are now included then the closed loop system becomes a sixth order system. The states of the augmented system are

\[ x_{cc}^{sd}(k) = \begin{bmatrix} i_1(k) \\ i_2(k) \\ v_c(k) \\ m(k) \\ p_i(k) \\ q_i(k) \end{bmatrix} \]

To find out the closed loop pole locations, modulation index \( m(k) \) is expressed in terms of system states as in (27). The closed loop characteristic matrix is given by (28).

\[
\begin{align*}
\Phi_{sd}^{cc} &= \begin{bmatrix} [\Phi_{sd}]_{4\times4} \\ -K_i [B_k]_{2\times1} \\ 0_{2\times3} \\ [A_k]_{2\times2} \end{bmatrix}, \\
\Gamma_{sd}^{cc} &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T.
\end{align*}
\]

For sampling period \( T = 200 \mu s \) the system is at stability limit when \( K_{pr} = 0 \) and becomes unstable for increased value of \( K_{pr} \). This can be observed from the pole locations variation shown in Fig. 4(b). Hence it would not be possible to operate the inverter with LCL filter with current control alone for such low \( \frac{F_{sw}}{f_{res}} \).
The discrete domain state space model of inverter with LCL filter along with PR controller for fundamental current control and ADRI method for active damping of filter resonance is of eighth order. The states of the closed loop system are $x^{ccp}_{sd}(k) = [i_1(k) \ i_2(k) \ v_c(k) \ m(k) \ p(k) \ q(k) \ p_i(k) \ q_i(k)]^T$.

$$x^{ccp}_{sd}(k + 1) = \Phi^{ccp}_{sd} x^{ccp}_{sd}(k) + \Gamma^{ccp}_{id} m(k)$$  \hspace{1cm} (29)
TABLE II
PARAMETERS OF LCL FILTER.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverter side inductor</td>
<td>$L_1$</td>
<td>4 mH</td>
</tr>
<tr>
<td>Winding resistance of $L_1$</td>
<td>$R_1$</td>
<td>0.70 Ω</td>
</tr>
<tr>
<td>Grid side inductor</td>
<td>$L_2$</td>
<td>4 mH</td>
</tr>
<tr>
<td>Winding resistance of $L_2$</td>
<td>$R_2$</td>
<td>0.70 Ω</td>
</tr>
<tr>
<td>Total filter capacitance</td>
<td>$C_f$</td>
<td>16 μF</td>
</tr>
<tr>
<td>Corner frequency of filter</td>
<td>$f_{res}$</td>
<td>890 Hz</td>
</tr>
</tbody>
</table>

Where,

\[ \Phi_{sd}^{ccp} = \begin{bmatrix} [\Phi_{sd}]_{4 \times 4} & [0]_{4 \times 4} \\ \end{bmatrix}, \quad \Gamma_{id}^{ccp} = \begin{bmatrix} [0]_{3 \times 1} \\ 1 \\ [0]_{4 \times 1} \end{bmatrix}, \]

\[ A_{RI} = \begin{bmatrix} [A_{kc}]_{2 \times 2} & [0]_{2 \times 2} \\ [0]_{2 \times 2} & [A_{k}]_{2 \times 2} \end{bmatrix}, \quad B_{RI} = \begin{bmatrix} [0]_{2 \times 2} & K_s[B_k]_{2 \times 1} & [0]_{2 \times 1} \\ -K_s[B_k]_{2 \times 1} & [0]_{2 \times 2} & [0]_{2 \times 1} \end{bmatrix} \]

(30)

The state space model of the inverter system with LCL filter, PR current control and ADRI in open loop is given by (29) and (30). The sub-matrices of $A_{RI}$ and $B_{RI}$ can be expressed using (13) and (23). The modulation index $m(k)$ is related to system states $x_{sd}^{ccp}$ by (31) and (32).

\[ m(k) = m_{pr}(k) - m_{ad}(k) = Mx_{sd}^{ccp}(k) \]

(31)

\[ M = \begin{bmatrix} -K_sD_k & 0 & K_g[K_v]_{1 \times 2} & 0 & K_g[C_k]_{1 \times 2} \end{bmatrix} \]

(32)

The modulation signal generated from the current controller and ADRI, shown in Fig. 2, is given as input to the VSI to obtain the closed loop model.

\[ \Phi_{sd}^{ccp} = \Phi_{sd}^{ccp} + [\Gamma_{id}^{ccp}]M \]

(33)

The closed loop characteristic matrix $\Phi_{sd}^{ccp}$ for the overall system of VSI with LCL filter, ADRI and PR current control is given by (33). By selecting different values of $R_v$ and $K_{pr}$ variation of eigenvalues of $\Phi_{sd}^{ccp}$ can be observed. This would give indication of relative stability and damping to poles of the closed loop system. The pole location variations of $\Phi_{sd}^{ccp}$ for different values of $K_{pr}$ and for sampling frequency $f_c = 5$ kHz is shown in Fig. 5. The $\times$ symbol indicates the pole locations when $K_{pr} = 0$ and $R_v = 4.9$ Ω. The pole locations indicated by $\Diamond$ symbol are when $K_{pr} = 0.2$ and $R_v = 4.9$ Ω.

**Choice of $K_{pr}$ and $K_{pr}$**: Here the gains obtained during the study of ADRI without the influence of current loop has been taken as the starting point. Similar to the previous case $K_{pr}$ has been varied by keeping the ratio $K_{pr}$ constant. Here value of $K_{pr}$ has been selected such that the closed loop system poles are sufficiently within the unit circle so that system stability is ensured.

**G. Effect of grid inductance on closed loop pole locations**:

Due to presence of grid inductance ($L_g$) resonant frequency $\omega_{res}$ changes from the designed value. The method suggested in [4] adopted for the design of the LCL filter with $L_1 = L_2$. Hence for values of $L_g$ from 0 to $\infty$, resonance frequency would remain within the range $\omega_{res} \in \left[ \sqrt{\frac{2}{L_1C_f}}, \sqrt{\frac{1}{L_1C_f}} \right]$. The effect of grid inductance ($L_g$) on closed loop pole location, particularly regarding stability, can be observed by considering the grid side inductor as $L_2 + L_g$, instead of $L_2$, and modifying the state space representation of the LCL filter system as given in (7).

The variation in eigenvalues of the characteristic matrix $\Phi_{sd}^{ccp}$ obtained for the case where both current control and ADRI is present in grid connected mode of operation, due to different values of grid inductance is shown in Fig. 6. Here the $\Diamond$ marked point is the chosen operating point for $K_{pr} = 0.2$ and $K_{pr}$ $= 500$. At this point $L_g = 0$ mH and this is indicated in Fig. 5 also. However Fig. 6 shows the locus for pole location variations starting at $\Diamond$ marked points and moving towards $\times$ marked points. The variation is plotted for values of $L_g$ ranging from 0 to 40 mH. Here the $\times$ marked points indicate the pole locations when $L_g = 11$ mH. This indicates that ADRI would lead to stable operation even with a wide range of grid impedance.
TABLE III
PARAMETERS OF PR CURRENT CONTROLLER AND ADRI. ($V_{dc} = 300$ V)

<table>
<thead>
<tr>
<th>$F_{sw}$</th>
<th>$\zeta_{RI}$</th>
<th>$f_0$</th>
<th>$K_f$</th>
<th>$R_v$</th>
<th>$K_{pr}$</th>
<th>$K_{ir}$</th>
<th>$K_{pr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5 kHz</td>
<td>0.4</td>
<td>890 Hz</td>
<td>44</td>
<td>4.9 Ω</td>
<td>0.2</td>
<td>500</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 7. Resonance frequency damping when both PR current control and ADRI are in use at $F_{sw} = 2.5$ kHz. Here $V_{dc} = 300$ V, $K_{ir} = 0.2$, $K_{pr} = 500$ and $R_v = 4.9$ Ω. (a) Both PR current controller and ADRI are working $L_g = 0$ mH. (b) Instability caused due to disabling ADRI. (c) Both PRC and ADRI are working with $L_g = 11$ mH. Scaling : X-axis→ 300 Hz/div (Frequency) and 10 ms/div (Time); Y-axis→ 50 V/div (Voltage) and 5 A/div (Current).

V. EXPERIMENTAL RESULT

Experiments were conducted to validate the theoretical understanding about the proposed damping method on a laboratory built 3 phase 4 wire two level inverter with LCL filter. The dc bus midpoint is utilised for the neutral wire. The single phase equivalent circuit diagrams are shown in Fig. 2. The inverter is operated as a STATCOM supplying reactive power to grid. Under this condition the effect of current control on damping alone and in combination with the proposed ADRI method, as well as effect of grid inductance variation has been experimentally observed for low value of $\frac{F_{sw}}{f_{res}}$. The control algorithm has been implemented using 16 bit fixed point arithmetic in an ALTERA Cyclone-II FPGA board. The parameters of LCL filter are given in Table-II. The control gains of PR current control and RISF for implementing ADRI are given in Table-III.

The results on damping performance for the case when both PR controller and ADRI are operating at $F_{sw} = 2.5$ kHz are shown in Fig. 7. The combined effect enables the system to operate in a stable manner, as shown in Fig. 7(a). If ADRI is disabled from operation the system becomes unstable, as shown in Fig. 7(b). Such a behaviour is as expected from the analysis in section-IV-D. This shows that for low ratio of $\frac{F_{sw}}{f_{res}}$ the PR current controller alone can not make the system operate in a stable manner but a combined operation of PR controller and ADRI can achieve stable operation. Fig. 7(c) shows the operation of the system when $L_g$ of value 11mH is added to emulate weak grid. Under this situation also the system is able to operate in stable manner and provides damping at the resonance frequency.
VI. CONCLUSION
In this paper a RISF based active damping of filter resonance called ADRI is proposed. Recent studies show stability concern regarding operation of grid connected VSI with LCL filter for low $\frac{f_{res}}{f_{sw}}$ ratio. Proposed ADRI scheme uses a high pass RISF structure to estimate the resonant frequency capacitor current through filter capacitance by sensing the capacitor voltage and further the RISF output is used for active damping. The modelling and analysis of ADRI includes all delay effects in the system. Hence additional delay compensation is not required. In this paper the analysis has been done for 3 cases at low $\frac{f_{res}}{f_{sw}}$ ratio. They are (a) VSI with LCL filter and ADRI but no current control, (b) VSI with LCL filter and PR current control but no active damping and (c) VSI with LCL filter, PR current control and ADRI. The analysis show that at low $\frac{f_{res}}{f_{sw}}$ ratio of 2.8 the system can not be stabilised with only PR current control. However along with proposed method of ADRI, the same VSI with LCL filter and PR current controller system can be operated in a stable manner. The analysis also show that the above system can work in a stable manner under wide range of grid inductances. All analysis and proposed design are validated through experiments.

REFERENCES
