El 216 COMPUTER VISION

LECTURE 06: IMAGE FEATURES

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- Earlier we looked at the **geometry** and **radiometry** of image formation
- In this lecture we shall look at **image features**
- Image features are low-level processing (early vision)

What's a feature?

- Encodes some meaningful information from an image
- Features can be *local* or *global* properties
- Features need to be detectable
- Notion of meaningful is context and application dependent
- Too vague and general?

What's a feature then?

- Points
- Straight Lines
- Parametric Curves
- Edges/Contours
- Texture
- Regions (superpixels)
- Global features

Above list is hierarchical but not exhaustive

Edge Detection



What's an edge?

Edges are pixels at or around which the image values undergo a sharp variation

Trucco & Verri, p. 69

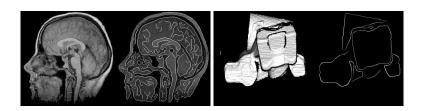
Edge Detection



What's an edge?

- A rapid change in brightness level
- Perceptual boundary between regions
- Edge strength and shape can vary
- Detection and localisation are key issues
- Need to counter effects of noise
- Lots of issues, well understood

Edge Detection



Why are we interested in edges?

- Often correspond to boundaries between regions
- Basic elements for further processing stereopsis, calibration, motion
- Grouping of edges results in meaningful percepts
- · Line drawings are succinct summaries











What are corners?

- Corners are point features
- Simple geometry
- Matchable across images
- How?

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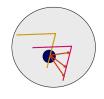




- Corners are point features
- Simple geometry
- Matchable across images
- How? Patches
- All patches equally good?

Szeliski 2nd Edition







What are corners?

- Corners are point features
- Simple geometry
- Matchable across images
- All patches equally good?
- Discuss feature description later

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What are corners?

- Edges are easy to define, hard to compute
- Corners are easy to compute
- An image corner need not have a physical interpretation
- Corners occur where patterns of intensities represent a perceptual corner





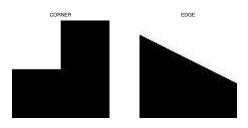


Why corners?

- Corner points represent sharp changes/discontinuties
- Corners are ubiquitous
- Corners are "relatively" stable to viewpoint variation
- Corners can be matched across images
- Corners can be easily tracked across a video sequence
- Vision problems can be easily defined and modelled using point features
- Multiview geometry of point features is well developed
- Computations using point features is efficient

Uses of corners?

- · Geometrically simple and well-defined
- Succinct representation of information
- Efficient computations
- Attach features to corners (matching, discrimination)
- Point matches for geometry
- Image retrieval (similar ones)
- Comparing image similarity
- Fundamental in computer vision systems



So what are corners?

- An edge represents intensity variation in one direction
- Intensity variation in orthogonal directions is much lower
- A corner represents intensity variation in both X and Y directions
- For image f(x, y), $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for corners are large

Notion of large $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ suggests an approach

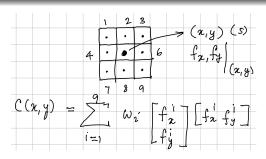
Structure Tensor

Denote $f_x = \frac{\partial f}{\partial x}$ and $f_y = \frac{\partial f}{\partial y}$

$$C = w_G(\sigma) * \begin{bmatrix} f_x^2 & f_x f_y \\ f_x f_y & f_y^2 \end{bmatrix} = \begin{bmatrix} \widehat{f_x^2} & \widehat{f_x} \widehat{f_y} \\ \widehat{f_x} \widehat{f_y} & \widehat{f_y^2} \end{bmatrix}$$

- Matrix is known as *local structure matrix* or *tensor*
- Encodes the structural variation of intensities around a point
- Corner-ness can be easily defined using matrix C
- The local structured is spatially smoothed using a Gaussian mask (denoted by hat)

Image Corners



Structure Tensor

- Aggregate information over $N \times N$ neighbourhood of (x, y)
- Size of *N* ?
- Weighting of individual contributions
- $w_i = \frac{1}{N^2}$ (Uniform)
- $w_i \propto \exp\{-\frac{1}{\sigma^2}((x-x_0)^2+(y-y_0)^2)\}$ (Gaussian)
- Form of w_i not very important
- Careful computation of (f_x, f_y) is essential. Why?

Image Corners

$$C = \left[egin{array}{cc} \widehat{f_x}^2 & \widehat{f_x}\widehat{f_y} \ \widehat{f_x}\widehat{f_y} & \widehat{f_y}^2 \end{array}
ight]$$

Properties

- For a single point, C is rank-1
- The structure emerges out of local averaging
- Matrix C is symmetric
- Matrix C is positive-definite
- Use these properties to detect corners

Image Corners

- Symmetry of $C \Rightarrow C$ is diagonalisable via a rotation U
- $C = UDU^T$ (Proof?)
- **Exercise**: What happens to f_x and f_y under rotation ? C ?
- Should we care about the rotation? Why?
- Diagonalised matrix will be of the form

$$D=C^{'}=\left[egin{array}{cc} \lambda_{1} & 0 \ 0 & \lambda_{2} \end{array}
ight]$$

• Positive-definite means the eigen-values are non-negative Wlog assume $\lambda_1 \geq \lambda_2 \geq 0$

Harris Corner Detector

- Proposed in 1988
- Quite popular till recent improvements
- Defines a measure of corner strength
- Detection using a threshold
- Uses neighbours to discard weak corners

Local Structure Matrix: Interpretation

- You may wonder why the given structure matrix C
- Consider the local auto-correlation function of the image
- For a shift of $(\Delta x, \Delta y)$ we have

$$R(\Delta x, \Delta y) = \sum_{W} \left[I(x, y) - I(x + \Delta x, y + \Delta y) \right]^{2}$$

• Taking Taylor series for a small shift $(\Delta x, \Delta y)$ we have

$$I(x + \Delta x, y + \Delta y) \approx I(x, y) + \begin{bmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$$I(x + \Delta x, y + \Delta y) \approx I(x, y) + \begin{bmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$$\Rightarrow R(x, y) = \sum_{W} \left(\begin{bmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \right)^{2}$$

Rearranging, we have

$$\begin{array}{rcl} R(x,y) & = & \displaystyle \sum_{W} \left[\begin{array}{ccc} \Delta x & \Delta y \end{array} \right] \left[\begin{array}{ccc} \sum_{W} I_{x}^{2} & \sum_{W} I_{x} I_{y} \\ \sum_{W} I_{x} I_{y} & \sum_{W} I_{y}^{2} \end{array} \right] \left[\begin{array}{ccc} \Delta x \\ \Delta y \end{array} \right] \\ \Rightarrow C & = & \left[\begin{array}{ccc} \sum_{W} I_{x}^{2} & \sum_{W} I_{x} I_{y} \\ \sum_{W} I_{x} I_{y} & \sum_{W} I_{y}^{2} \end{array} \right] \end{array}$$

Interpretation of C matrix

- Both λ_1 and λ_2 small 'flat' image function
- One λ large and the other small Auto-correlation function is ridge shaped Image intensity changes in one direction only This is the behaviour of an edge
- Both λ are large sharp fall in auto-correlation in any direction characteristic of a corner

Corner Strength

Defined as

$$H(x,y) = |C| - \alpha Tr(C)^{2}$$

where Tr() is the trace of a matrix.

This form of H means we do not need to explicitly compute $\,\lambda_1$ and $\,\lambda_2$

• For diagonalised matrix C we have

$$H(x,y) = \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2$$

$$H(x,y) = \lambda_1^2 (\kappa - \alpha (1 + \kappa)^2)$$

where
$$\kappa = \frac{\lambda_2}{\lambda_1}$$

• Corner detected when $H(x, y) \ge H_0$ (threshold)



Analysis

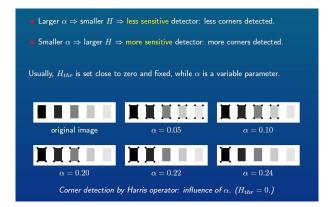
Assuming H > 0 we have

$$0 \le \alpha \le \frac{\kappa}{(1+\kappa)^2} \le 0.25$$

For small κ we have

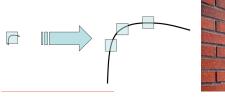
$$H \approx \lambda_1^2(\kappa - \alpha), \alpha \lesssim \kappa$$

$$H(x,y) = \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2$$



Taken from slides of Dmitrij Csetverikov

Refer to slides on **Harris Corner Detector** linked on page for this lecture

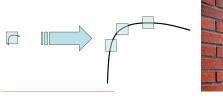




Role of Perspective Projection

- Scale of Harris corner detection?
- Multiple scales in same image?
- Only scale?
- Perspective plays a role
- What do we want?

adapted from K. Graumann; stock.adobe.com

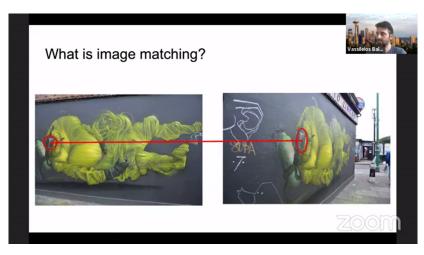


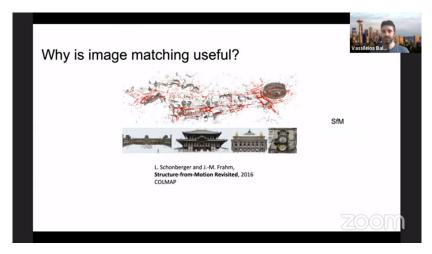


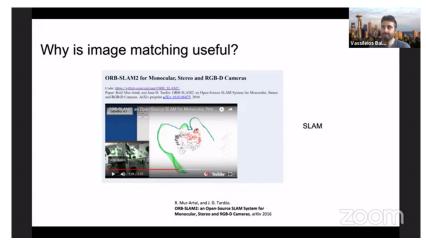
Role of Perspective Projection

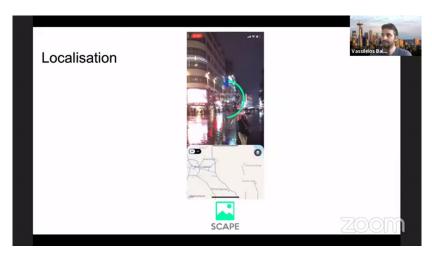
- Scale of Harris corner detection?
- Multiple scales in same image?
- Only scale?
- Perspective plays a role
- What do we want?
- Geometric Invariance

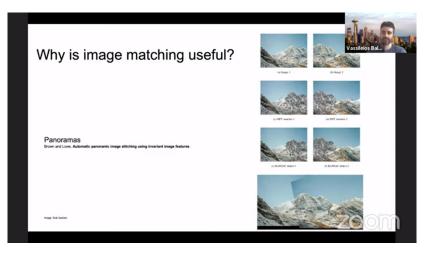
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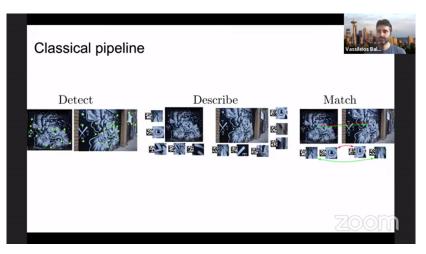














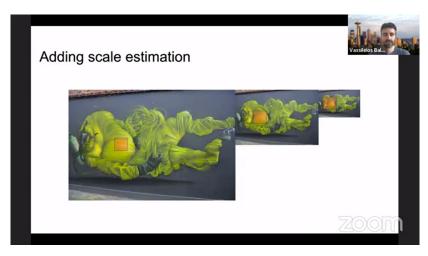
The classical image matching pipeline

Image A



- Step 1 Detection: Choose "interesting" points
- Step 2 Description: Convert the points to a suitable mathematical representation (descriptor)
- Step 3 Matching: Match the point descriptors between the two images





Refer to slides on SIFT linked on page for this lecture

Issues not addressed

- SIFT is *scale* invariant
- SIFT used as generic feature representation
- Affine invariance achieved by other features
- Faster versions of SIFT using integral images etc. (SURF)
- Histogram of Oriented Gradients (HOG) feature used for detection
- Key bottleneck : Feature matching in high-dim (128 dim)
- k-d tree representation
- Approximate nearest neighbour search in high-dim
- Binary features
- Feature descriptor is $local \Rightarrow$ false matches
- Need outlier removal for geometry estimation
- Recent progress: End-to-end learning
- CVPR 2020 Tutorial: 'Local Features: From SIFT to Differentiable Methods'